JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 1, Number 3, Summer 1988

A DISCRETE GALERKIN METHOD FOR FIRST KIND INTEGRAL EQUATIONS WITH A LOGARITHMIC KERNEL

KENDALL E. ATKINSON

ABSTRACT. Consider the first kind integral equation

$$\int_S g(Q) \log |P-Q| \, dS(Q) = h(P), \quad P \in S$$

with S a smooth simple closed curve in the plane. A special Galerkin method with trigonometric polynomial approximants has been shown by other authors to converge exponentially when solving the above integral equation. In this paper, Galerkin's method is further discretized by replacing the integrals with numerical integrals. The resulting discrete Galerkin method is shown to converge rapidly when the curve S and the data h are smooth. The method is also equivalent to a discrete collocation procedure with trigonometric polynomial approximants.

1. Introduction. Consider the numerical solution of

$$\int_S g(Q) \log |P-Q| \, dS(Q) = h(P), \quad P \in S, \tag{1.1}$$

with S the boundary of a simply-connected planar region D. This equation arises in solving the Dirichlet problem for Laplace's equation on D, using either a direct or indirect boundary integral equation reformulation of the Dirichlet problem. For this mathematical development of (1.1), see **[6]** or any of many other sources on boundary integral equation reformulations of Laplace's equation.

In this paper, we consider the restricted case that S is a smooth boundary curve. For simplicity, assume S has a C^∞ parameterization

$$r(s) = (\xi(s), \eta(s)), \quad 0 \le s \le 2\pi, \tag{1.2}$$

This was written while the author was a Visiting Professor in the School of Mathematics of the University of New South Wales, Sydney, Australia. The author was supported by the University of New South Wales ARGS progam grant "Numerical analysis for integrals, integral equations, and boundary value problems".