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ERROR BOUNDS FOR INTEGRAL EQUATIONS ON THE HALF LINE

Dedicated to Professor Günther Hämmerlin
to commemorate his 60^{th} birthday

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ABSTRACT. We compare solutions of integral equations

$$x(s) - \int_0^\infty k(s,t)x(t)dt = y(s),$$

$$x_\beta(s) - \int_0^\beta k(s,t)x_\beta(t)dt = y(s).$$

The setting for the analysis is the space of bounded, continuous functions on $[0,\infty)$. Under reasonable hypotheses, there are unique solutions x and x_β such that $x_\beta \to x$ as $\beta \to \infty$, uniformly on any finite interval. The main purpose of this paper is to obtain computable bounds for the error $|x_\beta - x|$, particularly for certain classes of Wiener-Hopf operators and for compact perturbations of Wiener-Hopf operators.

1. Background and objectives. We are concerned with integral equations of the form

$$x(s) - \int_0^\infty k(s,t)x(t)dt = y(s), \quad 0 \le s < \infty, \tag{1.1}$$

and the corresponding finite-section equations

$$x_\beta(s) - \int_0^\beta k(s,t)x_\beta(t)dt = y(s), \quad 0 \le s < \infty. \tag{1.2}$$

Such equations arise in probability theory, wave propagation, and radiative transfer, amongst other fields.

The functions x, x_β and y are assumed to be bounded and continuous. The hypotheses on the kernel $k(s,t)$ are

H1 $$\sup_{s\ge 0} \int_0^\infty |k(s,t)|dt < 1,$$