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WEAKLY SINGULAR INTEGRAL OPERATORS AS MAPPINGS BETWEEN FUNCTION SPACES

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ABSTRACT. Weakly singular integral operators K are investigated as mappings between function spaces of the Hilbert-Sobolev type defined on Riemannian manifolds M_n with boundary ∂M_n. The results obtained from this analysis are applied to the determination of function spaces for which the Fredholm integral equation of the first kind, $Ku = f$, admits solutions, and conditions on these function spaces are studied for which the boundary value problem $Ku = f$ in $M_n, u = g$ on ∂M_n has meaning.

1. Introduction. Let Ω_n be a bounded and open subset of $\mathbf{R}_n$, lying on one side of its boundary. The boundary of Ω_n, denoted by $\partial\Omega_n$, will be considered to be an infinitely differentiable manifold of dimension $n-1$.

Let K be the weakly singular integral operator K defined on the Sobolev space $H^s(\Omega_n), s \in \mathbf{R}$, by

$$(Ku)(x) = \int_{\Omega_n} k(x,y)u(y)dy,$$

where $0 \leq \alpha < n$ and $k(x,y) = 1/|x-y|^\alpha$.

The main purpose of this paper is to establish properties of weakly singular integral operators K as mappings between function spaces of the Hilbert-Sobolev type, and apply them to the study of the boundary value problem:

$$Ku = f \text{ in } \Omega_n \tag{1.1}$$

$$u = g \text{ on } \partial\Omega_n. \tag{1.2}$$

The action of K on certain subspaces H of $H^s(\Omega_n)$ is characterized, and these subspaces are shown to be mapped by K into $H^p(\Omega_n), q <$