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AN ALTERNATIVE APPROACH TO ILL-POSED PROBLEMS

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ABSTRACT. An approach to ill-posed problems is presented in which the domain of the operator is enlarged rather than the range restricted. A topology is then introduced which makes the inverse operator continuous. This leads to a regularization procedure based on analytic representations. A number of examples are presented as well.

1. Introduction. In one standard kind of ill-posed problem a linear operator T with non-closed range must be inverted in order to solve an equation of the form

$$Tf = g. \tag{1.1}$$

The difficulty arises when g belongs to the closure of the range but not to the range itself. Then (1.1) has no exact solution but has at best only an approximate solution. Even this approximate solution may not be adequate in that it may not be close to an exact solution because of the lack of continuity of the operator T^{-1}.

In a number of problems important in applications, T is an integral operator and the problem (1.1) is one of solving an integral equation of the first kind.

Many procedures have been proposed and used in resolving such problems. They are concisely summarized in the recent article by Nashed [**3**]; a more detailed exposition may be found in the book by Tikhonov and Arsenin [**8**].

We shall not review the current literature on the subject but merely remark that many of the standard approaches involve restricting the range of the operator. In particular, in Tikhonov's regularization method, the range is restricted to the image of a compact set. In

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