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EXISTENCE AND CONVERGENCE RESULTS FOR INTEGRAL INCLUSIONS IN BANACH SPACES

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ABSTRACT. The existence of solutions is established for multivalued Volterra integral equations (integral inclusions) defined in a separable Banach space and governed by convex and nonconvex orientor fields. Also we prove a convergence result for such integral inclusions. In doing that we obtain some new interesting results about multifunctions, including a new set valued version of Fatou's lemma.

Introduction-preliminaries. Several problems in applied mathematics (control theory, mathematical economics, mechanics, etc.) involve various types of ambiguity, indeterminacy, or uncertainty (which in particular includes the impossibility of a comprehensive description of the dynamics of the system under consideration). This leads to mathematical models that involve differential and integral inclusions. In recent years the study of this more general class of equations has received considerable attention and many mathematicians have contributed interesting results, mostly in the direction of differential inclusions.

The main purpose of the present paper is to study the problem of existence of solutions for Volterra type integral inclusions defined in a separable Banach space. We prove two existence theorems; one for convex orientor fields and the other for nonconvex ones. Then we present a convergence result for the family of integral inclusions that we consider. The convergence property is one of the most important properties in differential and integral equations. As was shown by Strauss-Yorke [**20**], much of the fundamental theory of ordinary differential equations follows directly from a convergence theorem. In the process of obtaining that convergence result we also prove a multivalued version of Fatou's lemma that generalizes earlier results of Artstein [**1**] and Schmeidler [**19**] and which is interesting in its own because of its important potential applications in control theory and mathematical economics.

Let $(\Omega, \sum)$ be a measurable space and X a separable Banach space.