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THE CLASSICAL SOLUTIONS FOR NONLINEAR PARABOLIC INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper, we consider the solvability in the classical sense of a class of nonlinear one-dimensional integrodifferential equations of parabolic type. The motivation for studying this problem comes from the many physical models in such fields as heat transfer, nuclear reactor dynamics and thermoelasticity. One of the characteristics of this kind of equation is that the maximum principle is no longer valid in general. We combine the integral estimate method and Schauder estimate theory for a linear parabolic equation to derive an *a priori* bound for the solution of our nonlinear problem in the norm of the Banach space $C^{2+\alpha,1+\frac{\alpha}{2}}(\overline{Q}_T)$. The method of continuity then allows us to establish the global existence of the solution. For completeness, we also demonstrate the uniqueness and continuous dependence of the solution.

1. Introduction. Let $\overline{Q}_T = [0,1] \times [0,T]$ with $T > 0$ arbitrary. In this paper we consider a nonlinear integrodifferential initial-boundary value problem of finding a function $u(x,t) \in C^{2+\alpha,1+\frac{\alpha}{2}}(\overline{Q}_T)$ which satisfies:

$$(1.1)\quad u_t = a(x,t,u,u_x)u_{xx} + b(x,t,u,u_x) + \int_0^t c(x,\tau,u,u_x)d\tau \text{ in } Q_T,$$

$$(1.2)\qquad u(0,t) = f_1(t), \quad 0 \le t \le T,$$

$$(1.3)\qquad u(1,t) = f_2(t), \quad 0 \le t \le T,$$

$$(1.4)\qquad u(x,0) = u_0(x), \quad 0 \le x \le 1.$$

The motivation for studying (1.1)-(1.4) arises from a variety of physical and engineering problems (see [**13, 20, 21**], etc.). Considerable

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