JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 1, Number 2, Spring 1988

ON LINEAR INTEGRO-DIFFERENTIAL EQUATIONS OF BARBASHIN TYPE IN SPACES OF CONTINUOUS AND MEASURABLE FUNCTIONS

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ABSTRACT. This paper surveys several important properties of linear integro-differential equations (1) of Barbashin type (2), especially those related to the geometric structure of the underlying function space. In contrast to Barbashin's classical results, discontinuous data (e.g. kernel functions) are also allowed. After discussing several classes of suitable kernels, the resolvent operator (Cauchy function) generated by the operator (2) is described. Moreover, stability results for equation (1) are proved. Finally, representation formulas for the corresponding Green's function are given; a perturbed version of such formulas applies to averaging procedures of Bogolyubov-Krylov type.

This paper is concerned with the linear differential equation

$$\frac{du}{dt} = A(t)u \tag{1}$$

in Banach spaces of continuous or measurable functions over some interval $[a, b]$, where the operator $A(t)$ is given for $t \in J$ (a bounded or unbounded interval in $\mathbf{R}$) by

$$A(t)x(s) = c(t, s)x(s) + \int_a^b k(t, s, \sigma)x(\sigma)d\sigma. \tag{2}$$

Here and in what follows, $c(t, s)$ and $k(t, s, \sigma)$ are measurable real functions on $J \times [a, b]$ and $J \times [a, b] \times [a, b]$, respectively. Appropriate function spaces for the operator (2) are, for instance, the space $X = C[a, b]$ of continuous functions or the space $X = L_p[a, b]$ of p -summable functions ($1 \leq p \leq \infty$) over $[a, b]$; more generally, X may be an ideal space (see e.g. [50]) of measurable functions.

AMS *Subject Classifications*: Primary 45J05, Secondary 34B27, 34C29, 34D05, 34D20, 34E10, 34G10, 46E30, 47G05.