JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 1, Number 2, Spring 1988

NUMERICAL SOLUTIONS OF INTEGRAL EQUATIONS ON THE HALF LINE II. THE WIENER-HOPF CASE.

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ABSTRACT. Numerical approximation schemes of quadrature type are investigated for integral equations of the form

$$x(s) - \int_0^\infty \kappa(s-t)x(t)dt = y(s), \quad 0 \leq s < \infty.$$

The principal hypotheses are that κ is integrable, bounded, and uniformly continuous on R, and that x and y are bounded and continuous or, alternatively, bounded and uniformly continuous, on R^+. The convergence of numerical integration approximations is established, along with error bounds in some cases. The analysis involves the collectively compact operator approximation theory and a variant of that theory in which the role of compact sets is played by bounded uniformly equicontinuous sets of functions on R^+.

1. Introduction. Consider a Wiener-Hopf integral equation

$$x(s) - \int_0^\infty \kappa(s-t)x(t)dt = y(s), \qquad 0 \leq s < \infty, \tag{1.1}$$

where x and y are bounded and continuous on $[0,\infty)$, and κ is bounded, uniformly continuous, and integrable on $(-\infty,\infty)$. For example, $\kappa(u) = e^{-|u|}$ or $\kappa(u) = 1/(1+u^2)$.

Finite-section approximations for (1.1) are given by

$$x_\beta(s) - \int_0^\beta \kappa(s-t)x_\beta(t)dt = y(s), \qquad 0 \leq s < \infty, \tag{1.2}$$

for $\beta \geq 0$. Numerical integration yields discrete approximations $x_{\beta n}$ for x_β and hence for x. As an illustration, the rectangular quadrature rule gives

$$x_{\beta n}(s) - \frac{1}{n}\sum_{i=1}^{\beta n} \kappa(s-\frac{i}{n})x_{\beta n}(\frac{i}{n}) = y(s), \qquad 0 \leq s < \infty, \tag{1.3}$$