ABSTRACT. Numerical approximation schemes of quadrature type are investigated for integral equations of the form

\[ x(s) - \int_0^\infty \kappa(s-t)x(t)dt = y(s), \quad 0 \leq s < \infty. \]

The principal hypotheses are that \( \kappa \) is integrable, bounded, and uniformly continuous on \( \mathbb{R} \), and that \( x \) and \( y \) are bounded and continuous or, alternatively, bounded and uniformly continuous, on \( \mathbb{R}^+ \). The convergence of numerical integration approximations is established, along with error bounds in some cases. The analysis involves the collectively compact operator approximation theory and a variant of that theory in which the role of compact sets is played by bounded uniformly equicontinuous sets of functions on \( \mathbb{R}^+ \).

1. Introduction. Consider a Wiener-Hopf integral equation

\[ (1.1) \quad x(s) - \int_0^\infty \kappa(s-t)x(t)dt = y(s), \quad 0 \leq s < \infty, \]

where \( x \) and \( y \) are bounded and continuous on \([0, \infty)\), and \( \kappa \) is bounded, uniformly continuous, and integrable on \((-\infty, \infty)\). For example, \( \kappa(u) = e^{-|u|} \) or \( \kappa(u) = 1/(1 + u^2) \).

Finite-section approximations for (1.1) are given by

\[ (1.2) \quad x_\beta(s) - \int_0^\beta \kappa(s-t)x_\beta(t)dt = y(s), \quad 0 \leq s < \infty, \]

for \( \beta \geq 0 \). Numerical integration yields discrete approximations \( x_{\beta n} \) for \( x_\beta \) and hence for \( x \). As an illustration, the rectangular quadrature rule gives

\[ (1.3) \quad x_{\beta n}(s) = \frac{1}{n} \sum_{i=1}^{\beta n} \kappa(s - \frac{i}{n})x_{\beta n}\left(\frac{i}{n}\right) = y(s), \quad 0 \leq s < \infty, \]