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CONSTRUCTION OF A CLASS OF INTEGRAL MODELS FOR HEAT FLOW IN MATERIALS WITH MEMORY

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1. Introduction. In this paper we construct a simple class of models for heat conduction in materials with memory; initial-boundary value problems associated with these models will be discussed in a forthcoming paper. We concentrate on situations in which the heat flux depends on the temporal history of the temperature gradient (and possibly on the present value and the history of the temperature), but is independent of the present value of the temperature gradient. Our models are based on Gurtin and Pipkin's theory of heat conduction [**12**]. An important feature of this theory - one that is relevant to experimental studies of heat flow in certain materials at very low temperatures - is that it predicts finite speed of propagation for thermal disturbances. There are other theories of heat conduction in materials with memory [1] (cf., e.g. [**6, 15**]). However, the framework of Gurtin and Pipkin seems best suited to our purposes.

We limit our attention to the one-dimensional case in which the only nonzero component of the heat flux is its x-component, q; moreover, q and the absolute temperature $\theta > 0$ are functions of x and the time t. In addition, we assume that the material is homogeneous and has unit density. In the absence of deformation the law of balance of energy reduces to

$$\dot{e} + q_x = r, \tag{1.1}$$

where $e = e(x,t)$ is the (specific) internal energy and $r = r(x,t)$ is the external heat supply. A superposed dot indicates differentiation with respect to time, while a subscript x indicates spatial differentiation. Equation (1.1) must be supplemented with constitutive assumptions that characterize the particular type of material. Since we consider

[1] The results of [**6, 15**] permit (but do not require) the heat flux to depend on the present value of the temperature gradient. If the heat flux is sensitive to small changes in the present value of the temperature gradient then the speed of propagation is not finite.