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SMOOTHNESS RESULTS OF SINGLE AND DOUBLE LAYER SOLUTIONS OF THE HELMHOLTZ EQUATIONS

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ABSTRACT. In this paper, we prove the smoothness results of single and double layer solutions for Helmholtz's equation in two and three dimensions. For the most part, results on the differentiability of single and double layer solutions of Laplace's equation extend to the corresponding results for the Helmholtz equation.

1. Introduction. It is well known that smoothness results of an integral operator are closely related to the rates of convergence of the approximate numerical solutions to the true solution of the corresponding integral equation. Atkinson [**1, 2**] applied a particular Galerkin method to the Laplace equation and gave a complete convergence and error analysis. In [**4** or **5**], the author applied the same Galerkin method to the exterior Dirichlet problem for the Helmholtz equation in three dimensions. The convergence and error analysis of this required smoothness results of single and double layer potentials. These results are well known for Laplace's equation (see [**3**]), but the analogous results for Helmholtz's equation are not available. In this paper, we prove smoothness results of single and double layer solutions of the Helmholtz equation in two and three dimensions. For the most part, results on the differentiability of single and double layer solutions of Laplace's equation extend to the corresponding results for the Helmholtz equation.

2. Definitions. We first introduce the following definitions in $\mathbf{R}^3$ (see [**3**, p. 97]).

DEFINITION 2.1. Let a function $f(x,y,z) = f(M)$, defined in a region D, be bounded and possess bounded and continuous derivatives up to order $\ell(\ell \geq 0)$, and let the derivatives of order ℓ be Hölder continuous. Thus

$$(2.1) \qquad \left|\frac{\partial^p f}{\partial x^{p_1}\partial y^{p_2}\partial z^{p_3}}\right| < A, \quad \begin{pmatrix} p_1+p_2+p_3=p \\ p=0,1,2,\ldots,\ell \end{pmatrix},$$