JOURNAL OF INTEGRAL
EQUATIONS AND APPLICATIONS
Volume 1, Number 1, Winter 1988

COERCIVE ESTIMATES AND EXISTENCE OF SOLUTIONS FOR A MODEL OF ONE-DIMENSIONAL VISCOELASTICITY WITH A NON-INTEGRABLE MEMORY FUNCTION

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ABSTRACT. We consider a model equation for one-dimensional motions of viscoelastic materials with a non-integrable memory function. We prove existence of solutions to the initial value problem globally in time when the data are smooth and small, and locally in time when the data are smooth but large. The proof of existence is based on coercive estimates for the linearized problem. Such estimates exploit the singular nature of the memory function.

1. Introduction. We consider integrodifferential equations of the form:

$$u_{tt}(x,t) = g(u_x(x,t))_x + \int_{-\infty}^{t} m(t-\tau)h(u_x(x,t),u_x(x,\tau))_x d\tau + f(x,t),\ \ x\in[0,1], t>0, \tag{1}$$

under the following assumptions:

(i) The functions g and h are of class C^4 and C^5, respectively.

(ii) The function $tm(t)$ is integrable on $(0,\infty)$.

(iii) $g'(0)>0$, and $h(p,p)=0,\ h_{,1}(p,p) = -h_{,2}(p,p) > 0$ for every $p\in\mathbf{R}$. Here $h_{,i}$ denotes the derivative with respect to the ith argument.

(iv) $m\in W^{1,1}[t_0,\infty)$ for every $t_0>0$ and $m>0, m'<0$.

Equations of the form (1) can be used to model one-dimensional motions of viscoelastic materials in both shear and elongation (see, e.g., [**6**, Chapter I]). The kernel m is called the memory function. The assumptions (i)-(iv) are physically natural ones. The variable u represents the displacement. For simplicity, we shall confine our attention to homogeneous Dirichlet boundary conditions

$$u(0,t)=u(1,t)=0,\ \ t>0; \tag{2}$$