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VOLTERRA INTEGRAL INCLUSIONS IN BANACH SPACES

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ABSTRACT. We prove the existence of solutions for Volterra integral inclusions governed by a nonconvex valued multifunction, with values in a separable Banach space. Then we examine some special kinds of solutions (like extremal solutions and quasitrajectories). In doing that we also prove some other results of independent interests (like a generalization of a result of Kato on weak convergence in the Lebesgue-Bochner spaces L^p_X and a set valued version of Fatou's lemma).

1. Introduction. In several areas of applied mathematics, like control theory, mathematical economics, mechanics, etc., we encounter problems that involve various types of ambiguity, indeterminacy or uncertainty (which in particular includes the impossibility of a comprehensive description of the dynamics of the system). The evolution of such systems is then described by a multivalued equation (differential or integral). In recent years a number of papers have appeared concerning integral inclusions. In particular we mention the works Ragimhanov [**20**], Lyapin [**15**], Cuong [**5**], Angell [**1**] and the recent work of the author [**19**]. The first two are treating inclusions of the Hammerstein type, while the last three treat general convex valued Urysohn inclusions of the Volterra type (Angell [**1**] examines Volterra integral inclusions with delays). Integral inclusions (as well as differential inclusions) arise naturally in control theory when the method of deparametrization is applied and in the study of feedback control (see Aubin-Cellina [**2**]). A different application of integral inclusions can be found in two recent papers of Glashoff and Prekels [**9**], [**10**], who consider problems related to thermostatic regulation in which the heating devices controlling the temperature of the system are governed by a relay switch.

The purpose of this note is to investigate the existence of solutions for nonconvex valued Volterra integral inclusions defined in a separable Banach space and to examine some properties of the solutions.

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