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EFFECTIVE BOUNDS FOR THE SINGULAR VALUES OF INTEGRAL OPERATORS

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I. Introduction. In [**1, 11**] results on the asymptotic behavior of condition numbers of matrices arising in the numerical treatment of integral equations of the first kind are presented. These are partially based on a theorem of Chang [**2**], which provides information on the asymptotic properties of singular values of certain integral operators. Chang's paper is a natural outgrowth of a long sequence of investigations of eigenvalues of such operators (see, e.g., [**4, 8, 9**]).

From the viewpoint of the numerical analyst, the estimates in [**1, 11**], being asymptotic, are rather unsatisfactory. Much more valuable would be actual bounds. A careful examination of Chang's work [**3**] reveals that such bounds are available there, but at the cost of a great deal of labor. We present here a simplified version of the proof of Chang. The desired bounds are an immediate by-product.

In Section II we prove a result on singular values of products of operators. (This maybe extracted from theorems of Weyl and Horn [**6, 10**]. We provide an elementary proof.) In §III this result is applied to the integral operators of primary interest, but under a somewhat restrictive hypothesis. This assumption is completely removed in §IV, and the desired bounds are obtained. In the final section, the possible extension of results to more general integral operators is discussed. Some classical eigenvalue bounds are obtained in the Appendix.

II. A basic lemma. We consider integral operators K, L, and M where

$$K\bullet = \int_0^1 K(x,y)\bullet dy, \tag{2.1}$$

with similar representations for L and M. All kernels are assumed to be in L_2. Denote by κ_j the singular values of K, with λ_j and μ_j the

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