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THE DISCRETE GALERKIN METHOD FOR NONLINEAR INTEGRAL EQUATIONS (*)

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ABSTRACT. Let $\mathcal{K}$ be a completely continuous nonlinear integral operator, and consider solving $x = \mathcal{K}(x)$ by Galerkin's method. This can be written as $x_n = P_n\mathcal{K}(x_n), P_n$ an orthogonal projection; the iterated Galerkin solution is defined by $\tilde{x}_n = \mathcal{K}(x_n)$. We give a general framework and error analysis for the numerical method that results from replacing all integrals in Galerkin's method with numerical integrals. A special high order formula is given for integral equations arising from solving nonlinear two-point boundary value problems.

1. Introduction. Consider the problem of solving the nonlinear Urysohn integral equation

$$x(t) = \int_{\Omega} K(t, s, x(s))ds, \quad t \in \Omega. \tag{1.1}$$

Denoting this equation by

$$x = \mathcal{K}(x), \tag{1.2}$$

we assume that $\mathcal{K}$ is a completely continuous operator from an open set $D \subset L^{\infty}(\Omega)$ into $C(\Omega)$, with Ω a set in $\mathbf{R}^m$, some $m \geq 1$. We will analyze the use of the discretized Galerkin method to solve for the fixed points x^* of $\mathcal{K}$.

Let S_h denote a finite dimensional approximating subspace of $L^{\infty}(\Omega)$, with h the discretization parameter. The Galerkin method for solving (1.2) is to find the element $x_h \in S_h$ for which

$$(x_h, \psi) = (\mathcal{K}(x_h), \psi), \text{ all } \psi \in S_h. \tag{1.3}$$

This is a well-analyzed method with a large literature; for example, see Krasnoselskii (1964), Krasnoselskii-Vainikko, et al. (1972), and

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