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ON AN INTEGRAL EQUATION WITH A BESSEL FUNCTION KERNEL

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ABSTRACT. The integral equation

$$g(x) = \int_0^\infty J_x^2(y) f(y) dy, \quad x \geq 0,$$

which arises in certain problems in stereology, is solved for a wide class of input functions $g(x)$ using transform techniques. Practical sufficient conditions for the validity of the solution representation are given and illustrative examples are presented.

1. Introduction and formal analysis. Integral equations of the first kind are typically far more difficult to solve than those of the second kind. Exceptions occur in the case of difference kernels (see, for example, [**2**, pp. 301ff.], [**9**, pp. 364ff], [**10**]), product or quotient kernels [**3**, pp. 214ff], and when transform techniques are applicable. The problem-at-hand falls into this latter category.

Our interest is in finding the function $f(y)$ which satisfies the integral equation

$$g(x) = \int_0^\infty J_x^2(y) f(y) dy, \quad x \geq 0, \tag{1.1}$$

where $g(x)$ is a known function of the nonnegative real variable x and $J_x(y)$ designates the Bessel function of the first kind of order x and argument y. The importance of this equation in spatial statistics or stereology was first brought to the attention of the author by Eugene Church of DOA [**1**]. The squared Bessel function kernel and the appearance of the independent variable in (1.1) are a bit unusual. Nevertheless, an inversion formula for this integral equation does exist and can be derived using a procedure based upon analytic function theory and our knowledge of two familiar integral transforms.