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SOME EXTENSIONS OF THE ARITHMETIC-MEAN THEORY OF ROBIN'S INTEGRAL EQUATION FOR BODIES WITH VERTICES

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ABSTRACT. A form of Neumann's method of the arithmetic mean was used previously to obtain an existence theorem for the homogeneous case of Robin's integral equation in E^2 for a convex closed curve with the vertex. In the present paper, this theory is developed to provide two further results, one of them uniqueness, and it is then shown how both existence and uniqueness follow for the non-homogeneous case (relevant to the Neumann problem). Finally, it is shown how a parallel theory can be given, embracing both the homogeneous and non-homogeneous cases, for E^3, that is to say, for Robin's integral equation for a convex closed surface with a vertex.

1. Introduction. Robin's integral equation in E^2 for a function σ on a simple closed curve C, is a Fredholm equation whose homogeneous case is

$$\sigma(A) = \frac{1}{\pi} \oint \sigma(A') \frac{\cos x}{r} ds' \ (A, A' \in C, r = \vec{A}' A), \tag{1}$$

C being conveniently parametrized by arc-length s, and where x is the angle between the r-direction and the outward-normal one to C at A.

Along with (1), we consider the integral equation of the first kind,

$$K + 2 \oint \sigma(A') \log r ds' = 0, \tag{2}$$

where K is a constant.

Physically, in the context of electrostatics, (2) represents the fact that the potential due to the charge density σ on a cylindrical conductor with cross section C, takes the constant value K on the cylinder, and for a continuous solution, (1) follows from the constancy of the potential throughout the interior of C.