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UPPER BOUNDS OF DEPTH OF MONOMIAL IDEALS

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ABSTRACT. Let $J \subsetneq I$ be two ideals of a polynomial ring S over a field, generated by square free monomials. We show that some inequalities among the numbers of square free monomials of $I \setminus J$ of different degrees give upper bounds of $\mathrm{depth}_S I/J$.

Introduction. Let $S = K[x_1, \dots, x_n]$ be the polynomial algebra in n variables over a field K, $d \leq t$, two positive integers and $I \supsetneq J$, two square free monomial ideals of S such that I is generated in degrees $\geq d$, respectively J in degrees $\geq d+1$. By [**2**, Proposition 3.1] and [**4**, Lemma 1.1] $\mathrm{depth}_S I/J \geq d$. Let $\rho_t(I \setminus J)$ be the number of all square free monomials of degree t of $I \setminus J$.

Theorem 0.1 [**4**, Theorem 2.2]. *If $\rho_d(I) > \rho_{d+1}(I \setminus J)$, then $\mathrm{depth}_S I/J = d$, independently of the characteristic of K.*

The aim of this paper is to extend this theorem. Our Theorem 1.3 says that $\mathrm{depth}_S I/J = t$ if $\mathrm{depth}_S I/J \geq t$ and

$$\rho_{t+1}(I \setminus J) < \sum_{i=0}^{t-d} (-1)^{t-d+i} \rho_{d+i}(I \setminus J).$$

The proof of this theorem (similarly of [**4**, Theorem 2.2]) uses the Koszul homology (especially the rigidity property of the Koszul homology [**1**, Exercise 1.6.31]) which proves to be a very strong tool in this frame. If $t = d$, then our theorem is precisely Theorem 0.1 (a

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