EXTREMAL REES ALGEBRAS

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Dedicated to Jürgen Herzog for his numerous contributions to Commutative Algebra on the occasion of his 70th birthday.

ABSTRACT. We study almost complete intersection ideals whose Rees algebras are extremal in the sense that some of their fundamental metrics–depth or relation type–have maximal or minimal values in the class. The focus is on those ideals that lead to *almost* Cohen-Macaulay algebras, and our treatment is wholly concentrated on the nonlinear relations of the algebras. Several classes of such algebras are presented, some of a combinatorial origin. We offer a different prism to look at these questions with accompanying techniques. The main results are effective methods to calculate the invariants of these algebras.

1. Introduction. Our goal is the study of the defining equations of the Rees algebras $\mathbf{R}[It]$ of classes of almost complete intersection ideals when one of its important metrics, especially depth or reduction number, attains an extreme value in the class. We are going to show that such algebras occur frequently and develop novel means to identify them. As a consequence, interesting properties of such algebras have been discovered. We argue that several questions, while often placed in the general context of Rees algebra theory, may be viewed as subproblems in this more narrowly defined environment.

Let $\mathbf{R}$ be a Cohen-Macaulay local ring of dimension d, or a polynomial ring $\mathbf{R} = k[x_1, \dots, x_d]$ for k a field. By an *almost complete intersection*