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Dedicated to Jürgen Herzog on his 70th birthday

ABSTRACT. We prove upper and lower bounds for all the coefficients in the Hilbert polynomial of a graded Gorenstein algebra $S = R/I$ with a quasi-pure resolution over R. The bounds are in terms of the minimal and the maximal shifts in the resolution of R. These bounds are analogous to the bounds for the multiplicity found in [**9**] and are stronger than the bounds for the Cohen Macaulay algebras found in [**5**].

1. Introduction. Let $S = \bigoplus S_i$ be a standard graded k-algebra of dimension d, finitely generated in degree one. $H(S, i) = \dim_k S_i$ is the Hilbert function of S. It is well known that $H(S, i)$, for $i \gg 0$, is a polynomial $P_S(x)$, called the *Hilbert polynomial* of S. $P_S(x)$ has degree $d - 1$. If we write

$$P_S(x) = \sum_{i=0}^{d-1} (-1)^i e_i \binom{x + d - 1 - i}{x} = \frac{e_0}{(d-1)!} x^{d-1} + \cdots + (-1)^{d-1} e_{d-1},$$

then the coefficients e_i are called the *Hilbert coefficients* of S. The first one, e_0 called the multiplicity, is the most studied and is denoted by e.

If we write $S = R/I$, where R is the polynomial ring in n variables and I is a homogeneous ideal of R, then all these coefficients can be computed from the shifts in the minimal homogeneous free R-resolution $\mathbf{F}$ of S given as follows:

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