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STANDARD DECOMPOSITIONS IN GENERIC COORDINATES

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Dedicated to Jürgen Herzog on the occasion of his seventieth birthday.

1. Introduction. Throughout the paper, $S = k[x_1, \dots, x_c]$ is a polynomial ring over an infinite field k, graded with $\deg(x_i) = 1$ for each i. We consider a graded finitely generated S-module M.

Let $\mathcal{A}$ be a subset of the variables $\{x_1, \dots, x_c\}$. Set $k[\mathcal{A}] = k[x_i \mid x_i \in \mathcal{A}]$. We say that a homogeneous element $m \in M$ is $\mathcal{A}$-*standard* if the map

$$\begin{aligned} k[\mathcal{A}] &\longrightarrow M \\ 1 &\longmapsto m \end{aligned}$$

is a monomorphism. Let $m_1, \dots, m_s \in M$ and $\mathcal{A}_1, \dots, \mathcal{A}_s$ be subsets of the variables $\{x_1, \dots, x_c\}$. A direct sum of vector spaces

$$M = \bigoplus_{1 \le i \le s} k[\mathcal{A}_i]\, m_i$$

is called a *standard decomposition* of M if m_i is $\mathcal{A}_i$-standard for each i. We say that the decomposition is *nested* if the $\mathcal{A}_i$ are nested subsets of $\{x_1, \dots, x_c\}$, that is, for each i, j one of $\mathcal{A}_i, \mathcal{A}_j$ is contained in the other. Easy arguments using "prime filtrations" (these are filtrations of M whose quotients have the form S/P for various prime ideals P) show that every module admits a standard decomposition (see [**6**, Section 1].)

A well-known combinatorial conjecture of Richard Stanley [**9**, Conjecture 5.1] asserts that a multigraded finitely generated module M of depth d has a standard decomposition as above where the m_i are multihomogeneous elements and every $\mathcal{A}_i$ has at least d variables. The

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