MULTIPLICITIES ASSOCIATED TO
GENERALIZED SYMBOLIC POWERS

STEVEN DALE CUTKOSKY

Dedicated to Juergen Herzog on the occasion of his 70th birthday

1. Introduction. Suppose that R is a Noetherian local ring of dimension d and I, J are ideals in R. Let

$$I_n(J) = I^n : J^\infty = \bigcup_{i=1}^\infty I^n : J^i,$$

be the "nth symbolic power of I with respect to J."

In the introduction to paper [**7**] by Herzog, Puthenpurakal and Verma, the following interesting question is raised.

Let s be the limit dimension of family $I_n(J)/I^n$. When does

$$\lim_{n\to\infty} \frac{e_{m_R}(I_n(J)/I^n)}{n^{d-s}}$$

exist?

In this paper we review some results in [**7**] and give a very general answer to this question, using some recent results from [**3, 4**].

2. Notation. m_R will denote the maximal ideal of a local ring R. $Q(R)$ will denote the quotient field of a domain R. $\ell_R(N)$ will denote the length of an R-module N. $\mathbf{Z}_+$ denotes the positive integers and $\mathbf{N}$ the nonnegative integers. Suppose that $x \in \mathbf{R}$. $\lceil x \rceil$ is the smallest integer n such that $x \leq n$. $\lfloor x \rfloor$ is the largest integer n such that $n \leq x$.

We recall some notation on multiplicity from [**10**, Chapter VIII, Section 10], [**8**, Section V-2] and [**2**, Section 4.6]. Suppose that (R, m_R)