

GENERIC CIRCUITS SETS AND GENERAL INITIAL IDEALS WITH RESPECT TO WEIGHTS

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We would like to dedicate this paper to Jürgen Herzog, teacher and collaborator, to his ability for sharing his passion for Commutative Algebra with so many students all over the world.

ABSTRACT. We study the set of circuits of a homogeneous ideal and that of its truncations, and introduce the notion of generic circuits set. We show how this is a well-defined invariant that can be used, in the case of initial ideals with respect to weights as a counterpart of the (usual) generic initial ideal with respect to monomial orders. As an application we recover the existence of the generic fan introduced by Römer and Schmitz for studying generic tropical varieties. We also consider general initial ideals with respect to weights and show, in analogy to the fact that generic initial ideals are Borel-fixed, that these are fixed under the action of certain Borel subgroups of the general linear group.

0. Introduction. In the study of homogeneous ideals in a polynomial ring it is a standard technique to pass to initial ideals. Also, in order to work with a monomial ideal more closely related to a given homogeneous ideal I , i.e., with a monomial ideal which shares with I important numerical invariants other than the Hilbert function, one can choose to work in generic coordinates or, in other words, to consider a generic initial ideal of I with respect to some monomial order. Even though some of the ideas underlying the notion of generic initial ideal were already present in the works of Hartshorne [12] and Grauert [10], a proper definition as well as the study of some of its main properties is to be found only later in the work of Galligo [9], where characteristic zero

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