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STRUCTURE OF GENERAL IDEAL SEMIGROUPS OF MONOIDS AND DOMAINS

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ABSTRACT. Let H be a monoid (respectively, an integral domain) and r an ideal system on H. In this paper we investigate the r-ideal semigroup of H. One goal is to specify monoids such that their r-ideal semigroup possesses semigroup-theoretical properties, like almost completeness, π-regularity and completeness. Moreover, if H is an integral domain and $*$ a star operation on H, then we provide conditions on H such that the idempotents of the $*$-ideal semigroup are trivial or such that H is $\pi *$-stable.

0. Introduction. In 1961 Dade, Taussky and Zassenhaus [**9**] investigated the semigroup structure of the ideal class semigroup of one-dimensional Noetherian domains with a focus on non-principal orders in algebraic number fields. In the sequel, this paper seems to have fallen into oblivion. It was reconsidered and generalized by Halter-Koch [**17**], who put the results into the context of the structure theory of semigroups as presented in [**14**]. In recent times, starting with a paper by Zanardo and Zannier [**23**], the structure of the ideal class semigroup of an integral domain attracted the interest of several authors. In particular, the Clifford and Boolean properties of ideal class semigroups was investigated. First, this was done for valuation domains in [**7**]. In the sequel, Bazzoni provided a general theory, focused on Prüfer domains [**2**–**5**]. Among others, she proved that a Prüfer domain has a Clifford semigroup if and only if it has finite character, and she highlighted the connection with stable domains. Following Bazzoni, Kabbaj and Mimouni in [**20**–**22**] investigated the Clifford and Boolean properties of the t-ideal class semigroup. Among others, they characterized Prüfer v-multiplication domains with Clifford t-class semigroup and determined the structure of their constituent groups.

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