JOURNAL OF COMMUTATIVE ALGEBRA
Volume 4, Number 3, Fall 2012

PROJECTIVE STAR OPERATIONS ON POLYNOMIAL RINGS OVER A FIELD

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ABSTRACT. We consider the polynomial ring $S := K[X_0, \dots, X_n]$ over a field K and the rings $R_i := K[(X_0/X_i), \dots, (X_n/X_i)]$ for $0 \le i \le n$. We introduce the notion of a projective star operation on S and relate it to the classical star operations on the R_i's. We show that the projective Kronecker function ring $\mathrm{PKr}\,(S, \star)$ of S is the intersection of the Kronecker function rings $\mathrm{Kr}\,(R_i, \star_i)$, $0 \le i \le n$, where the $\star_i$'s are pairwise compatible e.a.b. star operations on the R_i's and $\star$ is a projective star operation on S built from the $\star_i$'s.

1. Introduction. Let R be an integral domain with quotient field F. Let $\mathfrak{F}(R)$ denote the set of nonzero fractional ideals of R. We recall that a star operation on R is defined as a mapping $\star : \mathfrak{F}(R) \to \mathfrak{F}(R)$, $I \mapsto I^\star$, such that for all $I, J \in \mathfrak{F}(R)$ and $x \in F \setminus \{0\}$:

($\star_1$) $R^\star = R$ and $(xI)^\star = xI^\star$;

($\star_2$) $I \subseteq I^\star$, and $I \subseteq J \Rightarrow I^\star \subseteq J^\star$;

($\star_3$) $I^{\star\star} := (I^\star)^\star = I^\star$.

A star operation $\star$ is called *endlich arithmetisch brauchbar* (in brief *e.a.b.*) if for any finitely generated $I, J, H \in \mathfrak{F}(R)$, $(IJ)^\star \subseteq (IH)^\star$ implies $J^\star \subseteq H^\star$. Given an e.a.b. star operation $\star$, the ring $\mathrm{Kr}\,(R, \star) := \{f/g : f, g \in R[X] \setminus \{0\}, C(f)^\star \subseteq C(g)^\star\} \cup \{0\}$, where $C(f)$ denotes the content of the polynomial $f(X)$, is called the *Kronecker function of R* with respect to $\star$. It is known that $\mathrm{Kr}\,(R, \star)$ is a Bézout domain (a domain for which every proper nonzero finitely generated ideal is principal) with quotient field $F(X)$ and such that $\mathrm{Kr}\,(R, \star) \cap F = R$ (for an overview on star operations and Kronecker function rings see [**7**, Section 32]).

2010 AMS *Mathematics subject classification.* Primary 13A15, 13A18, 16W50.
Keywords and phrases. Star operations, Kronecker function rings, graded rings.
The results in this article formed a part of the authors' theses.
Received by the editors on November 4, 2010, and in revised form on September 7, 2011.

DOI:10.1216/JCA-2012-4-3-387 Copyright ©2012 Rocky Mountain Mathematics Consortium