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SEMILOCAL FORMAL FIBERS OF PRINCIPAL PRIME IDEALS

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ABSTRACT. Let $(T, \mathfrak{m})$ be a complete local (Noetherian) ring, C a finite set of pairwise incomparable nonmaximal prime ideals of T, and $p \in T$ a nonzero element. We provide necessary and sufficient conditions for T to be the completion of an integral domain A containing the prime ideal pA whose formal fiber is semilocal with maximal ideals the elements of C.

1. Introduction. One way to better understand the relationship between a commutative local ring and its completion is to examine the formal fibers of the ring. Given a local ring A with maximal ideal $\mathfrak{m}$ and $\mathfrak{m}$-adic completion $\widehat{A}$, the formal fiber of a prime ideal $P \in \operatorname{Spec} A$ is defined to be $\operatorname{Spec}(\widehat{A} \otimes_A k(P))$, where $k(P) := A_P/PA_P$. Since there is a one-to-one correspondence between the elements in the formal fiber of P and the prime ideals in the inverse image of P under the map from $\operatorname{Spec} \widehat{A}$ to $\operatorname{Spec} A$ given by $Q \to Q \cap A$, we can think of $Q \in \operatorname{Spec} \widehat{A}$ as being in the formal fiber of P if and only if $Q \cap A = P$.

One fruitful way of researching formal fibers has been, instead of directly computing the formal fibers of rings, to investigate "inverse" formal fiber questions–that is, given a complete local ring T, when does there exist a local ring A such that $\widehat{A} = T$ and both A and the formal fibers of prime ideals in A meet certain prespecified conditions? One important result of this type is due to Charters and Loepp, who show in [**1**] that, given a complete local ring T with maximal ideal $\mathfrak{m}$ and $G \subset \operatorname{Spec} T$ where G is a finite set of prime ideals which are pairwise incomparable by inclusion, a local domain A exists such that $\widehat{A} = T$ and the formal fiber of the zero ideal of A is semilocal with maximal

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