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ON THE PRIME IDEAL STRUCTURE OF SYMBOLIC REES ALGEBRAS

S. BOUCHIBA AND S. KABBAJ

ABSTRACT. This paper contributes to the study of the prime spectrum and dimension theory of symbolic Rees algebra over Noetherian domains. We first establish some general results on the prime ideal structure of subalgebras of affine domains, which actually arise, in the Noetherian context, as domains between a domain A and $A[a^{-1}]$. We then examine closely the special context of symbolic Rees algebras (which yielded the first counterexample to the Zariski-Hilbert problem). One of the results states that if A is a Noetherian domain and p a maximal ideal of A, then the Rees algebra of p inherits the Noetherian-like behavior of being a stably strong S-domain. We also investigate graded rings associated with symbolic Rees algebras of prime ideals p such that A_p is a rank-one DVR and close with an application related to Hochster's result on the coincidence of the ordinary and symbolic powers of a prime ideal.

1. Introduction. All rings considered in this paper are integral domains and all ring homomorphisms are unital. Examples of finite dimensional non-Noetherian Krull (or factorial) domains are scarce in the literature. One of these stems from the generalized fourteenth problem of Hilbert (also called Zariski-Hilbert problem). Let k be a field of characteristic zero, and let T be a normal affine domain over k. Let F be a subfield of the field of fractions of T. Set $R := F \cap T$. The Hilbert-Zariski problem asks whether R is an affine domain over k. Counterexamples on this problem were constructed by Rees [**27**], Nagata [**24**] and Roberts [**28, 29**]. In 1958, Rees constructed the first counter-example giving rise to (what is now called) Rees algebras. In

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The second author is the corresponding author.
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