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MONOIDS OF MODULES OVER RINGS OF INFINITE COHEN-MACAULAY TYPE

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ABSTRACT. Given a one-dimensional analytically unramified local ring $(R, \mathfrak{m})$, let $\mathfrak{C}(R)$ denote the monoid of isomorphism classes of maximal Cohen-Macaulay R-modules (together with $[0]$) with operation given by $[M]+[N] = [M \oplus N]$. If R is complete, then the Krull-Remak-Schmidt property holds; i.e., direct-sum decompositions of finitely generated R-modules are unique. If R is not complete, then properties of the monoid $\mathfrak{C}(R)$ measure how far R is from having the Krull-Remak-Schmidt property. Using a list of ranks of indecomposable maximal Cohen-Macaulay modules over the $\mathfrak{m}$-adic completion of R, we give a description of the monoid $\mathfrak{C}(R)$ when R has infinite Cohen-Macaulay type. Under certain hypotheses we show that, for arbitrary integers s and t both greater than one, there exists a maximal Cohen-Macaulay R-module M such that $M \cong L_1 \oplus \cdots \oplus L_s$ and $M \cong N_1 \oplus \cdots \oplus N_t$ for indecomposable maximal Cohen-Macaulay R-modules L_i and N_j.

1. Introduction. Let R be a commutative ring, and let $\mathcal{C}$ be a class of R-modules closed under isomorphism, finite direct sums and direct summands. We say the *Krull-Remak-Schmidt property* holds for the class $\mathcal{C}$ if, whenever $M_1 \oplus M_2 \oplus \cdots \oplus M_s \cong N_1 \oplus N_2 \oplus \cdots \oplus N_t$ for indecomposable modules M_i, $N_j \in \mathcal{C}$, then

(1) $t = s$, and

(2) there exists a permutation σ of the set $\{1, \ldots, s\}$ such that $M_i \cong N_{\sigma(i)}$ for each $i \in \{1, \ldots, s\}$.

Over a complete local ring, the Krull-Remak-Schmidt property holds for the class of finitely generated modules (see [**16**, Theorem 5.20]). Many authors, including Evans [**6**, Section 1] and Wiegand [**18**, Sections 3 and 4], have produced examples of noncomplete local rings for which direct-sum decompositions of finitely generated modules are

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