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MACAULAY'S THEOREM FOR SOME PROJECTIVE MONOMIAL CURVES

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1. Introduction. Throughout this paper S stands for the polynomial ring $k[x_1, \dots, x_n]$ over a field k with the standard grading $\deg(x_i) = 1$ for $1 \le i \le n$. For any graded ideal J of S, the size of J is measured by the Hilbert function

$$\begin{aligned} h : \mathbf{N} &\longrightarrow \mathbf{N} \\ i &\longmapsto \dim_k J_i, \end{aligned}$$

where $\mathbf{N} = \{0, 1, 2, \dots\}$ and J_i is the vector space of all homogeneous polynomials in J of degree i. In 1927, Macaulay [**9**] proved that, for every graded ideal in S, there exists a lex ideal with the same Hilbert function. Since then, lex ideals have played a key role in the study of Hilbert functions: in 1966, Hartshorne [**5**] proved that the Hilbert scheme is connected, namely, every graded ideal in S is connected by a sequence of deformations to the lex ideal with the same Hilbert function; then in the 1990s, Bigatti [**1**], Hulett [**6**] and Pardue [**11**] proved that every lex ideal in S attains maximal Betti numbers among all graded ideals with the same Hilbert function.

It is interesting to know if similar results hold for graded quotient rings of the polynomial ring S. One important class of graded quotient rings over which Macaulay's theorem holds is the Clements-Lindström ring $S/(x_1^{c_1}, \dots, x_n^{c_n})$, where $c_1 \le \cdots \le c_n \le \infty$. In 1969, Clements and Lindström [**2**] proved that Macaulay's theorem holds over the ring $S/(x_1^{c_1}, \dots, x_n^{c_n})$, that is, for every graded ideal in $S/(x_1^{c_1}, \dots, x_n^{c_n})$, there exists a lex ideal with the same Hilbert function. In the case $c_1 = \cdots = c_n = 2$, the result was obtained earlier by Katona [**7**] and Kruskal [**8**]. Recently, Mermin and Peeva [**10**] raised the problem to find other graded quotient rings over which Macaulay's theorem holds.

Toric varieties, cf. [**3**], have been extensively studied in algebraic geometry. They are very interesting because they can be studied with

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