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SOME REMARKS ON MULTIPLICATION AND FLAT MODULES

MAJID M. ALI

ABSTRACT. The purpose of this work is to study some properties of multiplication modules and flat modules. We give some properties of multiplication modules that characterize arithmetical rings. We investigate Ohm type properties for multiplication and flat modules, and we also characterize F-modules and FGP-modules.

1. Introduction. Throughout this paper all rings are assumed commutative with identity and all modules are unital. Let R be a ring and M an R-module. Then M is called a *multiplication* module if every submodule N of M has the form IM for some ideal I of R, [**12**]. Note that $I \subseteq [N : M]$ and hence $N = IM \subseteq [N : M]M \subseteq N$, so that $N = [N : M]M$. If K is a multiplication submodule of M, then for all submodules N of M, $N \cap K = [(N \cap K) : K]K = [N : K]K$. If M is a finitely generated faithful multiplication R-module, then M is cancelation [**28**, Corollary 1 to Theorem 9], from which it follows that $[IN : M] = I[N : M]$ for all ideals I of R and all submodules N of M. If M is a faithful multiplication module, then M is locally either zero or isomorphic to R. Thus, finitely generated faithful multiplication modules are locally isomorphic to R. Let P be a maximal ideal of R, and let $T_P(M) = \{m \in M : (1 - p)m = 0 \text{ for some } m \in M\}$. Then $T_P(M)$ is a submodule of M. M is called *P-torsion* if $T_P(M) = M$. On the other hand, M is called *P-cyclic* provided there exist $m \in M$ and $q \in P$ such that $(1 - q)M \subseteq Rm$. El-Bast and P.F. Smith [**14**, Theorem 1.2] showed that M is multiplication if and only if M is P-torsion or P-cyclic for each maximal ideal P of R. A multiplication module M is locally cyclic and the converse is true if M is finitely

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