JOURNAL OF COMMUTATIVE ALGEBRA
Volume 3, Number 4, Winter 2011

MONOIDS OF TORSION-FREE MODULES OVER RINGS WITH FINITE REPRESENTATION TYPE

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ABSTRACT. Given a local ring R, we let $\mathcal{T}(R)$ denote the monoid of isomorphism classes of finitely generated torsion-free R-modules with operation $[M] + [N] = [M \oplus N]$. The main goal of this paper is to determine which monoids occur as $\mathcal{T}(R)$ for one-dimensional local ring-orders R with finite representation type. A byproduct of this investigation is a Krull-Remak-Schmidt theorem for finitely generated torsion-free modules over these rings.

1. Introduction and terminology. It is well known [**17**] that the Krull-Remak-Schmidt property holds for the class of all finitely generated modules over any complete local ring. That is, whenever $M_1 \oplus M_2 \oplus \cdots \oplus M_s \cong N_1 \oplus N_2 \oplus \cdots \oplus N_t$ with each M_i and N_j indecomposable, then (1) $s = t$ and (2) there exists some permutation σ of the set $\{1, 2, \ldots, t\}$ such that $M_i \cong N_{\sigma(i)}$ for each i. Beginning with Evans [**7**], many authors, including Wiegand [**19**], have produced examples of non-complete local rings over which direct sum decompositions of finitely generated modules can be non-unique. In [**8**], the class of generically free modules over all local ring-orders was considered. The Krull-Remak-Schmidt property almost always fails for this larger class of modules. Thus, we restrict our rings and restrict to the nicer class of torsion-free modules. In [**2**] the first author gave a Krull-Remak-Schmidt theorem for the class of all finitely generated modules, and then for the much smaller class of finitely generated torsion-free modules over a family of equicharacteristic one-dimensional local rings $(R, \mathfrak{m})$ with *finite representation type*–those rings having, up to isomorphism, only finitely many indecomposable torsion-free modules. These results hinge on a result of Levy and Odenthal [**16**] and a list of possible ranks of indecomposable modules over the $\mathfrak{m}$-adic completion of R. Recently, the authors [**3**] gave a complete list of all possible ranks of indecomposable torsion-free modules over arbitrary one-dimensional reduced rings with finite representation type. The goal of this current

Received by the editors on March 24, 2009, and in revised form on November 14, 2009.

DOI:10.1216/JCA-2011-3-4-439