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A QUASI-LOCAL HALF-FACTORIAL DOMAIN WITH AN ATOMIC NON-HALF-FACTORIAL INTEGRAL CLOSURE

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ABSTRACT. We construct a *half-factorial* quasi-local domain R, so that its integral closure $\overline{R} = R[t]$, where $t^2, t^3 \in R$, is *atomic but not half-factorial*; $\overline{R}$ equals the seminormalization of R. Moreover, $\overline{R}$ is a quasi-local domain of bounded factorization, and every element in $\overline{R}$ of zero R-boundary is a unit in $\overline{R}$.

0. Preliminaries. For background on half-factoriality see [**2**]. A *half-factorial monoid* M is an atomic monoid in which every two decompositions into atoms of a non-unit in M have the same length. A *half-factorial domain* is a domain R so that the monoid $(R^\bullet, \cdot)$ is half-factorial.

It is well known that, unlike factorial domains, a half-factorial domain is not necessarily integrally closed (see [**3**]). Thus Valentina Barucci asked the following question [**2**]:

Is the integral closure $\overline{R}$ of a half-factorial domain R necessarily half-factorial?

The answer is negative as shown by Coykendall [**4**], who constructed a half-factorial domain so that its integral closure is not atomic. Thus Coykendall raised the question whether $\overline{R}$ is half-factorial if the domain R is half-factorial and $\overline{R}$ is atomic.

In this note we answer this question in the negative. For an integral domain A that is contained in a DVR, we extend A to a half-factorial domain $\mathrm{Hf}(A)$ (Section 1). We show that if A satisfies certain properties, then $\overline{\mathrm{Hf}(A)}$ is not half-factorial. In Section 2 we construct an integral domain A satisfying the desired properties.

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