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# INFINITARY EQUIVALENCE OF $\mathbf{Z}_p$-MODULES WITH NICE DECOMPOSITION BASES

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ABSTRACT. Warfield modules are direct summands of simply presented $\mathbf{Z}_p$-modules or, alternatively, are $\mathbf{Z}_p$-modules possessing a nice decomposition basis with simply presented cokernel. They have been classified up to isomorphism by their Ulm-Kaplansky and Warfield invariants. Taking a model theoretic point of view and using infinitary languages we give here a complete model theoretic characterization of a large class of $\mathbf{Z}_p$-modules having a nice decomposition basis. As a corollary, we obtain the classical classification of countable Warfield modules. This generalizes results by Barwise and Eklof.

**1. Introduction.** The classical theorem by Ulm [**13**] states that two countable abelian $p$-groups are isomorphic if and only if their numerical invariants, the Ulm-Kaplansky invariants, coincide. For uncountable (abelian) $p$-groups this theorem is false, however, it was Hill [**4**] and Walker [**14**] who proved that it still holds for the class of totally projective groups. In fact, the class of totally projective abelian $p$-groups is the largest natural class of abelian $p$-groups such that every member is completely determined by its Ulm-Kaplansky invariants. Passing to general abelian groups it was then Warfield [**15**] who extended Ulm's theorem to the class of Warfield modules introducing new numerical invariants, the so-called Warfield invariants. Recall that a Warfield module is a direct summand of a simply presented $\mathbf{Z}_p$-module, where $\mathbf{Z}_p$ is the ring of integers localized at the prime $p$. Taking a completely different point of view, it was Szmielew [**12**] who first considered abelian groups model-theoretically. The usual axioms for abelian groups can be stated in the lower predicate

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