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ON SOCLE-REGULARITY AND SOME NOTIONS OF TRANSITIVITY FOR ABELIAN p-GROUPS

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ABSTRACT. In the present work the interconnections between various notions of transitivity for Abelian p-groups and the recently introduced concepts of socle-regular and strongly socle-regular groups are studied.

1. Introduction. Early work in the theory of infinite Abelian p-groups focused on issues such as classification by cardinal invariants. This led initially to the rich theory known now as Ulm's theorem and, in some sense, culminated in deep classification of the class of groups known variously as *simply presented, totally projective* or *Axiom* 3 groups. Such groups are, of necessity, somewhat special. On the other hand, there was also interest in properties of groups that were held by "the majority" of Abelian p-groups. Within this latter category, the extensive classes of transitive and fully transitive groups were prominent. Recently, the present authors introduced two new classes of p-groups which, respectively, properly contained the corresponding classes of transitive and fully transitive groups: these are the socle-regular and strongly socle-regular groups developed in [**3, 4**]. The present paper looks further at the interconnections between these classes and some other recent notions of transitivity.

Throughout, all groups will be additively written, reduced Abelian p-groups; standard concepts relating to such groups may be found in [**6, 10**]. We follow the notation of these texts but write mappings on the right. To avoid subsequent need for definitions of fundamental ideas, we mention that the *height* of an element x in the group G (written like $h_G(x)$) is the ordinal α if $x \in p^\alpha G \setminus p^{\alpha+1} G$ with the usual convention that $h(0) = \infty$. The *Ulm sequence* of x with

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