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COSET DIAGRAMS IN THE STUDY OF FINITELY PRESENTED GROUPS WITH AN APPLICATION TO QUOTIENTS OF THE MODULAR GROUP

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ABSTRACT. We look at some ways in which coset diagrams have been used to study quotients, subgroups and structure of finitely presented groups. Then we apply one of those known methods to get a partial answer to what happens when adding one relator to the modular group. We find that the language of such words making the resulting group isomorphic to S_3 is regular and that the language of words making the group infinite contains a subset that is a context sensitive language.

1. Introduction. In this paper we will look at how coset diagrams have been used in the study of finitely presented groups. Such diagrams, sometimes under the name *Schreier diagrams*, is a relatively old concept and seems to originate from work by Otto Schreier and Kurt Reidemeister in the 1920s. A description in more modern terms can be found in [**7**] from 1966. The interest has risen in the last decades as the possibilities to use diagram techniques in combination with mathematical software has improved. In the first part of this paper we will describe a couple of different applications of coset diagrams to the study of finitely presented groups. The aim of this part is not to introduce new ideas, but rather to clarify how a number of more specific applications are based on the same diagram technique. In the second part we will confine ourselves to a particular type of finitely presented groups, namely one-relator quotients of the modular group. These have been studied before by Conder in [**3**] where he described all such groups with third relator of length at most 24. The method used was to either prove finiteness and find the structure of the group using coset enumeration or, in the cases where the computations did not finish indicating an infinite group, constructing infinite transitive coset diagrams to prove infinitude. This was done to fill in a gap in an investigation by Tucker

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