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THREE-DIMENSIONAL MANIFOLDS, SKEW-GORENSTEIN RINGS AND THEIR COHOMOLOGY

JAN-ERIK ROOS

Dedicated to Ralf Fröberg and Clas Löfwall at their 65th birthdays.

ABSTRACT. Graded skew-commutative rings occur often in practice. Here are two examples: 1) The cohomology ring of a compact three-dimensional manifold. 2) The cohomology ring of the complement of a hyperplane arrangement (the Orlik-Solomon algebra). We present some applications of the homological theory of these graded skew-commutative rings. In particular, we find compact oriented 3-manifolds without boundary for which the Hilbert series of the Yoneda Ext-algebra of the cohomology ring of the fundamental group is an explicit transcendental function. This is only possible for large first Betti numbers of the 3-manifold (bigger than, or maybe equal to, 11). We give also examples of 3-manifolds where the Ext-algebra of the cohomology ring of the fundamental group is not finitely generated.

0. Introduction. Let X be an oriented compact 3-dimensional manifold without boundary. The cohomology ring $H = H^*(X, \mathbf{Q})$ is a graded skew-commutative ring whose augmentation ideal $\overline{H}$ satisfies $\overline{H}^4 = 0$. The triple (cup) product $x \cup y \cup z = \mu(x, y, z).e$, where e is the orientation generator of H^3, defines a skew-symmetric trilinear form on H^1 with values in $\mathbf{Q}$, i.e. a *trivector*, and conversely, according to a theorem of Sullivan [**36**] any such form comes in this way from a 3-manifold X (not unique) whose cohomology algebra can be reconstructed from μ since by Poincaré duality $H^2 \simeq (H^1)^*$. In the more precise case when H^* is also a Poincaré duality algebra, i.e., the cup product $H^1 \times H^2 \to H^3$ is nondegenerate, it follows that H^* is a Gorenstein ring (cf. Section 1 below). Such Gorenstein rings will be studied here. Any 3-manifold M can be decomposed in a unique way

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