

SPLITTING ALGEBRAS AND GYSIN HOMOMORPHISMS

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ABSTRACT. We give three algebraic constructions of splitting algebras of monic polynomials with coefficients in arbitrary commutative rings with a unit, and of the corresponding Gysin homomorphisms.

1. Introduction. We give three algebraic constructions of splitting algebras of monic polynomials with coefficients in arbitrary commutative rings with a unit, and of the corresponding Gysin homomorphisms. The first construction of splitting algebras is by induction, and is well known (see, e.g., [2, 5, 12, 13, 15–17, 21, 22]), the second is the most natural, via symmetric polynomials, and is a variant of the method used by Bourbaki [2], and the third, using the division algorithm over polynomial rings, was indicated to us by A. Thorup (University of Copenhagen), and has not appeared in the literature. We believe that all of the constructions of Gysin homomorphisms are new. Many of the ideas and methods used in our constructions evolved during our cooperation with Throup. We are also thankful to Bengt Ek (The Royal Institute of Technology, Stockholm) and Michael Shapiro (Michigan State University, East Lansing) for the elegant proofs of some auxiliary results. For other treatments of Gysin homomorphisms see [1, 3, 4, 6–8, 10, 12, 13, 15–20].

The importance of having several constructions of the same object and of the homomorphisms between them is that we in this way shed light on the area, but also that we by comparing the constructions can obtain interesting polynomial identities. We have not performed these comparisons here, but leave them to the reader.

Splitting algebras and Gysin homomorphisms appear in many different parts of mathematics. Best known is their usefulness to the cohomology theory of flag schemes, and in particular in Schubert calculus

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