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BETTI NUMBERS OF SOME SEMIGROUP RINGS

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ABSTRACT. We compute the Betti numbers of all semigroup rings R corresponding to numerical semigroups of maximal embedding dimension. A description, in terms of the generators of S, precisely in which degrees the nonzero graded Betti numbers occur is given. We show that for arithmetic numerical semigroups of maximal embedding dimension, the graded Betti numbers occur symmetrically in two respects.

1. Introduction and preliminaries. By a *numerical semigroup* we mean a submonoid S of $\mathbf{N}$ such that $\mathbf{N}\setminus S$ is finite. $\mathbf{N}$ is understood to be the set of non-negative integers $\{0, 1, 2, \dots\}$. It is well known that such a semigroup is finitely generated, that is, it consists of all non-negative integer combinations of some minimal generating set $\{s_0, s_1, \dots, s_n\}$.

In this paper we only consider numerical semigroups and hence by semigroup we will always mean numerical semigroup. We denote a semigroup S minimally generated by the elements $\{s_0, s_1, \dots, s_n\}$ by $S = \langle s_0, s_1, \dots, s_n\rangle$. If nothing else is said we assume $s_0 < \cdots < s_n$. Here the number $n+1$ is the *embedding dimension* of S and is denoted by $e(S)$. The number s_0 is called the *multiplicity* of S and is denoted by $m(S)$. We always have $e(S) \le m(S)$ and, if $e(S) = m(S)$, we say that S has *maximal embedding dimension*.

Given a semigroup $S = \langle s_0, \dots, s_n\rangle$ and a field k, consider the *semigroup ring* $R = k[S]$. This is the k-algebra $k[t^s\,;\, s \in S]$, t an indeterminate, defined by

$$t^s \cdot t^{s'} = t^{s+s'}, \quad s, s' \in S.$$

If A is the polynomial ring $k[x_0, \dots, x_n]$, we can define a homomorphism

$$A \xrightarrow{\phi} R$$

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