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INTEGRAL CLOSURE AND OTHER OPERATIONS ON MONOMIAL IDEALS

VERONICA CRISPIN QUIÑONEZ

ABSTRACT. In this paper we give a characterization of integrally closed monomial ideals in two variables. The notion of x- and y-tight ideals is introduced. We also present a monomial version of a result of Watanabe on chains of integrally closed monomial ideals. Using the developed techniques, we prove results about the quadratic transforms and products in classes of monomial ideals, and describe multiplication in two classes of ideals in an integral domain.

1. Introduction and preliminaries. Let R be a polynomial (localized) ring or a power series ring. The maximal (irrelevant) ideal is denoted by $\mathfrak{m}$. An ideal I is called $\mathfrak{m}$-primary if its radical $\sqrt{I} = \mathfrak{m}$. An ideal is *simple*, if it is not a product of two proper ideals.

A power product is an element $x_1^{a_1} \cdots x_n^{a_n}$. If a monomial ideal is written as $I = \langle x^{a_i} y^{b_i} \rangle$, we usually assume that the generators are ordered in such a way that $a_i < a_{i+1}$ and $b_i > b_{i+1}$.

An element $r \in R$ is said to be *integral* over an ideal I in R, if r satisfies an *equation of integral independence*

$$r^l + a_1 r^{l-1} + \cdots + a_{l-1} r + a_l = 0 \text{ where } a_j \in I^j.$$

The *integral closure* of I is defined as the set of all elements in R which are integral over I. This closure is denoted by $\overline{I}$.

If $r \in R$, then $o(r) = \max(l | r \in \mathfrak{m}^l)$. The order of an ideal $I \subset R$ is defined as $o(I) = \min\{o(r) | r \in I\}$. The least number of generators of I is denoted by $\mu(I)$.

In Appendix 5 of [**14**], which is based on [**13**], it is proved that in a two-dimensional regular local ring the product of integrally closed ideals is integrally closed. This is not the case in a three-dimensional

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