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DECOMPOSITIONS OF IDEALS INTO IRREDUCIBLE IDEALS IN NUMERICAL SEMIGROUPS

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ABSTRACT. It is proved that each ideal I of a numerical semigroup S is in a unique way a finite irredundant intersection of irreducible ideals. The same result holds if "irreducible ideals" are replaced by "**Z**-irreducible ideals." The two decompositions are essentially different and, if $n(I)$ and $N(I)$ respectively are the number of irreducible or **Z**-irreducible components, it is $n(I) \leq N(I) \leq e$, where e is the multiplicity of S. However, if I is a principal ideal, then $n(I) = N(I) = t$, where t is the type of S.

1. Introduction. In one of her famous papers, [**8**], Emmy Noether shows that each proper ideal of a Noetherian ring admits a representation as an irredundant intersection of finitely many irreducible ideals. Such a representation is not unique, but the number of components is uniquely determined by the ideal. The present paper deals with numerical semigroups, which are mathematical objects much simpler than Noetherian rings. So it is not surprising that the results of decomposition of an ideal as intersection of irreducible ideals are stronger. Such a decomposition in fact, if irredundant, is unique, as can be easily proved (cf. Theorem 3.3). On the other hand, the irreducibility of ideals in rings can also be considered in terms of fractional ideals. In a ring R, with total ring of quotients Q, a fractional ideal J is said to be Q-irreducible if it is not the intersection of two fractional ideals properly containing it (cf. [**5**]). The concepts of ideal and fractional ideal in rings have natural correspondences in numerical semigroups. In fact, similarly to Q-irreducible fractional ideals, **Z**-irreducible relative ideals in a numerical semigroup can be defined. It turns out that a relative ideal of a numerical semigroup S is **Z**-irreducible if and only if it is of the form $z + \Omega$, for some $z \in \mathbf{Z}$, where Ω is the canonical ideal of S. Theorem 4.4 shows that a relative ideal of a numerical semigroup S is in a unique way an irredundant intersection of **Z**-irreducible ideals. However, given an ideal I of S, $I \subset S$, the two decompositions as irre-

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