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SEQUENTIALLY COHEN-MACAULAYNESS VERSUS PARAMETRIC DECOMPOSITION OF POWERS OF PARAMETER IDEALS

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ABSTRACT. This paper proves that parametric decomposition of powers of parameter ideals characterizes, under some additional conditions, sequentially Cohen-Macaulayness in modules together with a certain *good* property of corresponding systems of parameters.

1. Introduction. Let A be a commutative ring, and let $\underline{x} = x_1, x_2, \dots, x_d \in A$ $(d > 0)$ be a system of elements in A. Let M be an A-module. For each integer $n \geq 1$ we put

$$\Lambda_{d,n} = \Big\{\alpha = (\alpha_1, \alpha_2, \dots, \alpha_d) \in \mathbf{Z}^d \mid \alpha_i \geq 1 \quad \text{for } 1 \leq \forall i \leq d,\ \sum_{i=1}^{d} \alpha_i = d + n - 1\Big\}.$$

Let $Q = (x_1, x_2, \dots, x_d)$ and $Q(\alpha) = (x_1^{\alpha_1}, x_2^{\alpha_2}, \dots, x_d^{\alpha_d})$ for each $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_d) \in \Lambda_{d,n}$. Then we say that the system $\underline{x}$ satisfies condition (PD) for M, or all the powers of Q have parametric decomposition in M, if the equality

$$Q^n M = \bigcap_{\alpha \in \Lambda_{d,n}} Q(\alpha) M$$

holds true for all $n \geq 1$. Here we notice that $Q^n M \subseteq Q(\alpha) M$ for every $\alpha \in \Lambda_{d,n}$ and so, condition (PD) requires the inclusion $Q^n M \supseteq \bigcap_{\alpha \in \Lambda_{d,n}} Q(\alpha) M$ only.

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