JOURNAL OF COMMUTATIVE ALGEBRA
Volume 2, Number 1, Spring 2010

GORENSTEIN PROJECTIVE DIMENSION WITH RESPECT TO A SEMIDUALIZING MODULE

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Dedicated to the memory of Colleen Kilker

ABSTRACT. We introduce and investigate the notion of G_C-projective modules over (possibly non-Noetherian) commutative rings, where C is a semidualizing module. This extends Holm and Jørgensen's notion of C-Gorenstein projective modules to the non-Noetherian setting and generalizes projective and Gorenstein projective modules within this setting. We then study the resulting modules of finite G_C-projective dimension, showing in particular that they admit G_C-projective approximations, a generalization of the maximal Cohen-Macaulay approximations of Auslander and Buchweitz. Over a local ring, we provide necessary and sufficient conditions for a G_C-approximation to be minimal.

1. Introduction. Over a Noetherian ring R, Foxby [**9**], Golod [**10**] and Vasconcelos [**19**] independently initiated the study of semidualizing modules (under different names): a module C is semidualizing if $\operatorname{Hom}_R(C,C) \cong R$ and $\operatorname{Ext}_R^{\geqslant 1}(C,C) = 0$. Examples include the rank 1 free module and a dualizing (canonical) module, when one exists. Golod [**10**] used these to define G_C-dimension, a refinement of projective dimension, for finitely generated modules. The G_C-dimension of a finitely generated R-module M is the length of the shortest resolution of M by so-called totally C-reflexive modules; see Definition 4.1. Motivated by Enochs and Jenda's extensions in [**7**] of Auslander and Bridger's G-dimension [**2**], Holm and Jørgensen [**12**] have extended this notion to arbitrary modules over a Noetherian ring. The current paper provides a unified and generalized treatment of these concepts, in part by removing the Noetherian hypothesis. The tools

2010 AMS *Mathematics subject classification.* Primary 13D02, 13D05, 13D07, 13D25, 18G20, 18G25.
Keywords and phrases. Gorenstein dimensions, G_C-dimensions, G_C-approximations, proper resolutions, strict resolutions, totally C-reflexives, complete resolutions, complete PC-resolutions, semidualizing modules, Bass classes, C-projectives.
Received by the editors on August 21, 2007, and in revised form on December 29, 2008.

DOI:10.1216/JCA-2010-2-1-111