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FREE RESOLUTIONS OF SOME EDGE IDEALS OF SIMPLE GRAPHS

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ABSTRACT. The goal of this paper is to study the structure of the minimal free resolutions associated to a class of squarefree monomial ideals by using the one-to-one correspondence between squarefree quadratic monomial ideals and the set of all simple graphs. In [**6**], Hà and Van Tuyl demonstrated an inductive procedure to construct the minimal free resolution of certain classes of edge ideals. We will provide a simplified and more constructive proof of this result for the class of simple graphs containing a vertex of degree 1. Furthermore, by using the graphical structure of a tree, we provide a comprehensive description of the Betti numbers associated to the corresponding edge ideal along with providing an implementation of this graphical method coded in Python for use in SAGE. Furthermore, for specific subclasses of trees, we will generate more precise information including explicit formulas for the projective dimensions and Castelnuovo-Mumford regularity corresponding to the associated edge ideals. Although the methods discussed to study the edge ideals of simple graphs rely on the graph having a vertex of degree 1, we show how these methods and results can be used to gain information about the edge ideals of graphs that do not have a vertex of degree 1 by studying the class of edge ideals associated to cycles.

1. Introduction. By a *graph* G, we mean a vertex set $V_G = \{x_0, \dots, x_n\}$ along with a set of edges $E_G \subset V_G \times V_G$. Moreover, if $\{x_i, x_j\} \in E_G$ we will say x_i and x_j are *connected by an edge.* A graph is called *simple* if it is undirected and contains no loops or multiple edges. In this paper, we will restrict ourselves to the class of simple graphs, thus enabling a one-to-one correspondence between the set of simple graphs and the set of square-free quadratic monomial ideals.

$$\left\{\begin{array}{c}\text{Square-free quadratic}\\ \text{monomial ideals}\\ I \subset S = k[x_0, \dots, x_n]\end{array}\right\} \overset{1:1}{\longleftrightarrow} \left\{\begin{array}{c}\text{Simple graphs } G\\ \text{on } n+1 \text{ vertices}\end{array}\right\}.$$

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