

A CLASS OF LOCAL NOETHERIAN DOMAINS

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ABSTRACT. In this paper, we construct factorial domains with a given specific completion. The material is based on Nishimura's paper [12], which is based on the work of Rotthaus in [17], Ogoma in [15] and Heitmann in [6].

1. Introduction. In this paper we present a method of constructing local Noetherian rings which has been very fruitful over the past 25 years in the construction of various examples and counterexamples in commutative algebra. After being introduced first in 1979 this method has been simplified, modified, extended, and generalized by many authors resulting in numerous papers published and unpublished. Our goal here is to describe this method comprehensively so that the construction of additional examples is merely a matter of choosing the right ideal in a polynomial ring and plugging in the appropriate equations. The presentation is guided by unpublished notes of Jun-ichi Nishimura. We are grateful to Nishimura for allowing us to make use of his notes in this paper. Nishimura's notes also include a large number of new and previously unknown examples of Noetherian rings which are not included here.

Over the past 60 years important examples of Noetherian local rings have been constructed using so called non-standard methods. Non-standard methods are methods which go beyond the standard ways of constructing Noetherian rings like extensions of finite type, localization, completion, and Henselization. These non-standard methods can roughly be divided into three classes. The first class of examples, called here the Akizuki-Nagata method, goes back to at least Akizuki [1]. It was used and extended by Nagata [10] in his famous example of a normal local Noetherian domain whose completion is not a domain. The methods presented in this paper can be understood as a modification of the Akizuki-Nagata method. The local Noetherian rings

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