

ON TENSOR PRODUCTS OF RINGS AND EXTENSION CONJECTURES

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ABSTRACT. We show that a commutative local version of a conjecture of Tachikawa holds over a Cohen-Macaulay tensor product of rings provided it holds over the rings themselves.

1. Introduction. The purpose of this note is to broaden the context in which some homological conjectures hold.

The following commutative local version of a conjecture of Tachikawa has been of interest recently (see [4, 9]).

Conjecture (Tachikawa). *Let A be a Cohen-Macaulay local ring. If A has a canonical module ω and $\text{Ext}_A^i(\omega, A) = 0$ for all $i > 0$, then A is Gorenstein, i.e., $\omega = A$.*

Implying this conjecture of Tachikawa is another conjecture, which is a commutative local version of one of Auslander and Reiten, and which has also been of interest (see [1, 2, 8, 9]).

Conjecture (Auslander-Reiten). *Let A be a commutative Noetherian local ring, and let M be a finitely generated A -module. If $\text{Ext}_A^i(M, M \oplus A) = 0$ for all $i > 0$, then M is free.*

Suppose that (R_1, \mathfrak{m}_1) and (R_2, \mathfrak{m}_2) are commutative local rings which are essentially of finite type over the same field k . We also assume that k is the common residue field of both R_1 and R_2 . In this note we are concerned with what we call the *local tensor* R of R_1 and R_2 , this being the localization of $R_1 \otimes_k R_2$ at the maximal ideal $\mathfrak{m} := \mathfrak{m}_1 \otimes_k R_2 + R_1 \otimes_k \mathfrak{m}_2$. The main point here is that properties of vanishing homology and cohomology for modules over the local tensor

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