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COTORSION PAIRS INDUCED BY DUALITY PAIRS

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ABSTRACT. We introduce the notion of a duality pair and demonstrate how the left half of such a pair is "often" covering and preenveloping. As an application, we generalize a result by Enochs et al. on Auslander and Bass classes, and we prove that the class of Gorenstein injective modules, introduced by Enochs and Jenda, is covering when the ground ring has a dualizing complex.

1. Introduction. What is now known as semi-dualizing modules were studied more than 25 years ago under other names by, e.g., Foxby [**15**] (PG-modules of rank one), Golod [**17** (suitable modules) and Vasconcelos [**33**] (spherical modules). As a common generalization of the notion of a semidualizing module and that of a dualizing complex, in the sense of Hartshorne [**18**], Christensen [**7**] introduced in 2001 the notion of a semidualizing complex, cf. (1.5).

Avramov and Foxby [**1**] and Christensen [**7**] demonstrated how a semidualizing complex C over a commutative Noetherian ring R gives rise to two important classes of R-modules, namely the so-called Auslander class A_0^C and Bass class B_0^C, cf. (1.6). Semidualizing complexes and their Auslander and Bass classes have caught the attention of several authors, but this paper is motivated by a result of Enochs and Holm [**10**], for which we prove the following generalization in Theorem 3.2.

Theorem A. *Let R be a commutative Noetherian ring, and let C be a semidualizing complex of R-modules. Then the following conclusions hold*:

(a) $(\mathsf{A}_0^C, (\mathsf{A}_0^C)^\perp)$ *is a perfect cotorsion pair, in particular, the class* A_0^C *is covering. Furthermore,* A_0^C *is preenveloping.*

(b) *The class* B_0^C *is covering and preenveloping.*

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