

**QUASI-SOCLE IDEALS
IN LOCAL RINGS
WITH GORENSTEIN TANGENT CONES**

SHIRO GOTO, SATORU KIMURA, NAOYUKI MATSUOKA
AND TRAN THI PHUONG

ABSTRACT. Quasi-socle ideals, that is, the ideals I of the form $I = Q : \mathfrak{m}^q$ in a Noetherian local ring (A, \mathfrak{m}) with the Gorenstein tangent cone $G(\mathfrak{m}) = \bigoplus_{n \geq 0} \mathfrak{m}^n / \mathfrak{m}^{n+1}$ are explored, where $q \geq 1$ is an integer and \overline{Q} is a parameter ideal of A generated by monomials of a system x_1, x_2, \dots, x_d of elements in A such that (x_1, x_2, \dots, x_d) is a reduction of \mathfrak{m} . The questions of when I is integral over \overline{Q} and of when the graded rings $G(I) = \bigoplus_{n \geq 0} I^n / I^{n+1}$ and $F(I) = \bigoplus_{n \geq 0} I^n / \mathfrak{m} I^n$ are Cohen-Macaulay are answered. Criteria for $G(I)$ and $\mathcal{R}(I) = \bigoplus_{n \geq 0} I^n$ to be Gorenstein rings are given.

1. Introduction. This paper aims at a study of quasi-socle ideals in a local ring with the Gorenstein tangent cone. Our purpose is to answer Question 1.1, below, of when the graded rings associated to the ideals are Cohen-Macaulay and/or Gorenstein rings, estimating their reduction numbers with respect to minimal reductions.

Let A be a Noetherian local ring with the maximal ideal \mathfrak{m} and $d = \dim A > 0$. Let $Q = (x_1, x_2, \dots, x_d)$ be a parameter ideal in A , and let $q \geq 1$ be an integer. We put $I = Q : \mathfrak{m}^q$ and refer to those ideals as quasi-socle ideals in A . Then one can ask the following questions, which are the main subject of the researches [8, 9] and the present research as well.

Question 1.1. (1) Find the conditions under which $I \subseteq \overline{Q}$, where \overline{Q} stands for the integral closure of Q .

(2) When $I \subseteq \overline{Q}$, estimate or describe the reduction number $r_Q(I) = \min \{0 \leq n \in \mathbf{Z} \mid I^{n+1} = QI^n\}$

of I with respect to Q in terms of some invariants of Q or A .

2000 AMS *Mathematics subject classification.* Primary 13H10, 13A30, 13B22, 13H15.

Keywords and phrases. Quasi-socle ideal, regular local ring, Cohen-Macaulay ring, Gorenstein ring, associated graded ring, Rees algebra, Fiber cone, integral closure.

Received by the editors on April 30, 2008.

DOI:10.1216/JCA-2009-1-4-603 Copyright ©2009 Rocky Mountain Mathematics Consortium