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WHEN IS TIGHT CLOSURE DETERMINED BY THE TEST IDEAL?

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ABSTRACT. We characterize the rings in which the equality $(\tau I : \tau) = I^*$ holds for every ideal $I \subset R$. Under certain assumptions, these rings must be either weakly F-regular or one-dimensional.

1. Introduction. Test ideals play a major role in the theory of tight closure. The tight closure of arbitrary ideals is very difficult to compute, even in relatively simple rings, but the test ideal can be frequently computed, especially in Gorenstein rings. Moreover, test ideals encode geometric information about the nature of the singularity of the ring. We recall the definitions and basic facts.

Throughout this paper, $(R, \mathfrak{m})$ is a local domain of characteristic p. We denote positive integer powers of p by q.

Definition 1.1. Let $I \subset R$ be an ideal. We say that $x \in R$ is in the *tight closure*, I^*, of I if there is a $c \neq 0$ such that $cx^q \in I^{[q]} = (\{i^q | i \in I\})$. We say that I is *tightly closed* if $I = I^*$.

Definition 1.2. The *test ideal* τ is defined by

$$\tau = \bigcap_{I \subset R} (I : I^*),$$

where I runs over all ideals $I \subset R$.

The fact that $\tau \neq (0)$ is a highly nontrivial and important result.

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