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MAPS ON DIVISOR CLASS GROUPS INDUCED BY RING HOMOMORPHISMS OF FINITE FLAT DIMENSION

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ABSTRACT. Let $\varphi : A \to B$ be a ring homomorphism between Noetherian normal integral domains. We establish a general criterion for φ to induce a homomorphism $\mathrm{Cl}(\varphi) : \mathrm{Cl}(A) \to \mathrm{Cl}(B)$ on divisor class groups. For instance, this criterion applies whenever φ has finite flat dimension; this special case generalizes the more classical situations where φ is flat or is surjective with kernel generated by an A-regular element. We extend some of Spiroff's work on the kernels of induced maps to this more general setting.

Introduction

The divisor class group of a Noetherian normal integral domain A, denoted $\mathrm{Cl}(A)$, measures certain aspects of the factorization-theory of A. For instance, it is well-known that A is a unique factorization domain if and only if $\mathrm{Cl}(A)$ is trivial. For definitions and notation, consult the beginning of Section 1.

It is natural to investigate the transfer of such factorization properties between rings that are connected by a ring homomorphism. As part of such an investigation, one should find nontrivial classes of ring homomorphisms $\varphi : A \to B$ of Noetherian normal integral domains that induce group homomorphisms $\mathrm{Cl}(\varphi) : \mathrm{Cl}(A) \to \mathrm{Cl}(B)$. For instance, the flat ring homomorphisms have this property. Danilov [**7**, Prop. 1.1] shows that the natural surjection $B[\![T]\!] \to B$ also has this property, and Lipman [**14**, §0] extended this to any surjection of the form $A \to A/fA$, assuming that A and A/fA are both Noetherian normal integral domains.

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