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RINGS OF FINITE RANK AND FINITELY GENERATED IDEALS

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ABSTRACT. Here we provide examples of rings of minimum rank n for every positive integer n. We also introduce a tool that is used to count irreducible elements in finitely generated prime ideals in atomic domains.

1. Introduction. The concept of finite generation is prominent in commutative algebra. Indeed, Noetherian rings, where every ideal is finitely generated, form one of the richest and most fruitful classes of commutative rings with identity. In this paper we consider two classes of rings: the class of rings of finite rank, which is a restrictive subclass of Noetherian rings, introduced by Cohen [**1**]; the class of rings that satisfy the n-generator property, which is a restrictive subclass of Prüfer domains, introduced by Gilmer [**2**]. Both of these classes are defined below.

Let I be an ideal of R. We say that I is n-generated if it can be generated by a set of n elements. Using notation from [**3**], we denote by $\mu(I)$ the minimal number of generators of I. Additionally, we say that the ring R is of finite rank n if every ideal of R is n-generated. For convenience, we will say that a ring that is an element of the class of rings that are off finite rank n is M_n.

The concept of mininum rank will also prove useful. We say that the ring R has minimum rank n if it is M_n and contains an ideal I such that $\mu(I)$ is n. Following the notation from [**4**], we use $\mu_*(R)$ to denote the minimum rank of R. For an example, note that if R is a Dedekind domain, then $\mu_*(R)$ is 1 if $|\mathrm{Cl}(R)|$ is 1 and is 2 otherwise.

It is also natural to generalize these concepts to the realm of non-Noetherian domains. One could consider a ring to have the n-generator

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