JOURNAL OF COMMUTATIVE ALGEBRA
Volume 1, Number 3, Fall 2009

MINIMAL INTERSECTIONS AND VANISHING (CO)HOMOLOGY

DAVID A. JORGENSEN AND W. FRANK MOORE

ABSTRACT. We introduce a class of local Noetherian rings, which we call *minimal intersections*, and show that over such rings there exist classes of modules for which the derived functors Ext and Tor vanish non-trivially. This generalizes a well-known phenomenon of non-trivial vanishing of Ext and Tor for modules over complete intersections of codimension at least two.

1. Introduction. Let R be a commutative local Noetherian ring, and M and N finitely generated R-modules. In many cases the vanishing of all higher Ext and Tor can only occur in a trivial way. For instance, in [**9, 11, 12, 20**] it is shown that over hypersurfaces (which are codimension one complete intersections), Golod rings and Gorenstein rings of low codimension, the vanishing of all higher $\operatorname{Tor}_i^R(M,N)$ or $\operatorname{Ext}_R^i(M,N)$ implies that either M has finite projective dimension, or N has finite projective dimension (or finite injective dimension for Ext vanishing if R is not Gorenstein). This raises a question of the rarity of non-trivial vanishing of all higher homology and cohomology over local rings.

The most well-known class of local rings over which the vanishing of all higher Ext and Tor occurs non-trivially is that of complete intersections of codimension at least two (see, for example, [**14**, Theorem 3.1], and [**4**]). In this paper we isolate a property of complete intersections which enables non-trivial vanishing, and consider, more generally, local Noetherian rings having this property:

2000 AMS *Mathematics subject classification.* Primary 13D03.
Keywords and phrases. vanishing Ext, vanishing Tor.
The first author was partially supported by NSA Grant Number H98230-07-1-006.
Received by the editors on April 24, 2008, and in revised form on September 7, 2008.
DOI:10.1216/JCA-2009-1-3-507